Experimental Study on the Effect of Size Ratio and Number Percentage on Packing Density of Random Binary Packing of Particles

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(9MS)

“WANT A BAG OF MARBLES CONTAINING UNIVERSES...”
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1 Introduction

1.1 Background Information

When buying a bag of potatoes, should one full of equally sized potatoes or the one with size variations? Which bag would give the buyer more mass? When purchasing a carton of apples or oranges, should one buy the one regularly packed, or the one randomly filled? Would the one with size variation be cost-benefit? These are typical particle packing problems which can usually be generalised by studying packing of solid components to find an arrangement in which the components fill as large a proportion of the space as possible. The proportion of space filled by the components is called the density of the packing $\phi$, which is the ratio of the volume of the particles $V_p$ to the volume of the space $V_s$ filled by the particles:

$$\phi = \frac{V_p}{V_s} \quad (\text{a})$$

Particle packing is a phenomenon that exists everywhere in our life, from large scale the stars and galaxies packed in our universe, to the small scale of atom world. It is a topic widely encountered in science (colloids, granular media), engineering (catalyst carriers, food technology) and nature (the sand on Manly beach, or the gravel in our backyards). As such, it is not surprised that the study of particle packing can be traced back into history even since 17th century.

In 1611, Johannes Kepler conjectured that for equal spheres, the densest packing uses maximum $\frac{\pi}{3\sqrt{2}}$ (approximately 74\%) of the volume amongst both regular and irregular
packing \[^{1,2,3}\]. This well-known Kepler Conjecture attracted the attention of many scientists and mathematicians in history including Carl Friedrich Gauss \[^{4}\]. This maximum density is directly derived from packing patterns of either the “Face Centred Cubic” (FCC) or the “Hexagonal Close Packed” (HCP) structure \[^{5}\], which can be classified as regular packing, as shown in Figure 2 below:

![Figure 2. The FCC and HCP structure.](image)

Examples in FCC structure can be found in metals at room temperature, e.g. Aluminium, Copper, Gold, Lead, Nickel, Platinum, and Silver, whereas examples in HCP structure include Cadmium, Cobalt, Magnesium, Titanium (Alpha) and Zinc \[^{5}\]. A formal proof of the conjecture was finally considered completed by Hales et al in 2014 and 2015 \[^{6,7}\].

Comparing to the above mentioned regular packing, the so-called random close packing (RCP) can be built by placing spheres randomly into a container and then compressed until an irregular packing structure is formed with no more compression becoming possible \[^{8}\]. Figure 3 demonstrates a typical RCP structure obtained from a computer simulation \[^{9}\]. Recent investigation suggests that the highest possible packing density of RCP of mono-sized spheres is approximately 0.634 \[^{10}\].
Figure 3. An illustration of Random Close Packing (RCP) structure.

When size varies, packing of spherical components instantly becomes complicated, because both size ratio and population of the components disturb the packing structure, and thus result in change of packing densities. One extreme example is the Apollonian sphere packing, as shown in Figure 4. It is constructed in the principle that with any four spheres that are cotangent to each other, it is possible to introduce two more spheres that are cotangent to four of them and so on \cite{11}. This result in packing with higher density as illustrated below.

Figure 4. An illustration of a 3D Apollonian sphere packing

Yet, spheres are exceptions! Factors like shape of particles will further complicate the packing structure thus the packing densities dramatically \cite{12}. For example, although being
mono-sized, random packing of M&Ms, as shown in Figure 5 will results in a packing density of 0.68 [13], a denser packing structure surprisingly tighter than RCP of mono-sized spheres. On the other hand, the packing of toothpicks as shown in Figure 6 would only build a very loose random packing with much lower packing density, as one usually experiences when filling out green vegetation recycling bins with sticks and branches.

Figure 5 Packing of M&Ms [13]  Figure 6 A loose packing of toothpicks [14]

Forces including interparticle forces at different scales and conditions are also important in regard to packing structure and packing density. A weaker gravitational force, for example, contributes the loose packing structure of moon dust [15], whereas for most stars, strong gravity will create very dense and compact stellar remnants, also called compact stars [16]. On the other hand, interparticle forces e.g. capillary force, electrostatic forces and van der Wasls forces etc. are found dominate on packing of particles with size smaller than 100 µm [17, 18].

Table 1 below summarises the variables that may affect the packing of particles from both experimental observations and theoretical analysis [11].
Table 1. The variables which affect the packing of particles \cite{11}.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Experimental observation</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>I. Particle</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shape</td>
<td>*</td>
<td>**</td>
</tr>
<tr>
<td>Absolute size</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Size distribution</td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>Mass</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Elasticity</td>
<td>**</td>
<td>*</td>
</tr>
<tr>
<td>Resilience</td>
<td>**</td>
<td>*</td>
</tr>
<tr>
<td>Surface properties</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td><strong>II. Container</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shape</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Size</td>
<td>**</td>
<td>*</td>
</tr>
<tr>
<td>Elasticity</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Surface properties</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td><strong>III. Deposition</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intensity of deposition</td>
<td>**</td>
<td>*</td>
</tr>
<tr>
<td>Velocity of depositing</td>
<td>**</td>
<td>*</td>
</tr>
<tr>
<td>particles</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Method</td>
<td>*</td>
<td>x</td>
</tr>
<tr>
<td><strong>IV. Treatment after deposition</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vibratory compaction</td>
<td>**</td>
<td>*</td>
</tr>
<tr>
<td>Pressure compaction</td>
<td>**</td>
<td>*</td>
</tr>
<tr>
<td><strong>Note:</strong> x no evident; * qualitative evidence; ** quantitative evidence**</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As listed above, it is a challenge, if not entirely impossible, to regulate clearly the effect of the variables on packing structure and its density, especially when the “interaction” of these variables always exists in reality. As concluded by Oger et al \cite{19} that the form of packing structure depends on various numbers of parameters, many of which are difficult to measure or even define. It is often impossible to deduce the effect of each of all these variables on the packing particles. Thus, the study of a random packing of particles is often an art than science of the methods used to characterise particulate systems. Nowadays, as highlighted by the
intensive study of the packing of sands\textsuperscript{[20]}, for example, it is commonly accepted that under certain given conditions, i.e. same packing method, materials and container used etc., the packing density of particles is mainly determined by the size and shape distribution of the particle components.

In this study, we are interested in the effect of size ratio and relative population of the particles on the packing density of randomly packed binary systems.

1.2 Aim

To study the effect of size ratio and number percentage of small component on packing density of random binary packing of particles.

1.3 Hypothesis

- That the random mixing of different sizes particles would result in a change of packing density comparing to mixing of mono-sized spherical particles.
- That the larger the variation of the particle size, the higher the packing density would become.
- That for a given size ratio, there would be an optimum percentage in terms of the number of the small particles in the binary mixture that would result in the highest density of the packing.

2 Materials and Methods

2.1 Identification of Variables

2.1.1 Independent variables

- **Size ratio** between the radii of the small particle to that of the large particle:

\[ \eta = \frac{r_s}{r_l} \]
where η is the size ratio, rs is the radius of the small particle, and rt is the radius of the large particle.

According to the definition, η will have a value range of:

\[ 0 \leq \eta \leq 1 \]

- **Number Percentage** of the small particles to the total number of particles of the binary mixture:

\[
p_s = \frac{N_s}{N_{s+t}} \times 100\% = \frac{N_s}{N_s+N_t} \times 100\% \quad (c)
\]

where \( p_s \) is the number percentage of the small particles, \( N_s \) is the total number of small particles in the binary mixture, \( N_t \) is the total number of particles of the mixture, and \( N_{s+t} \) is the total number of large particles in the mixture.

According to the definition, \( p_s \) will have a value range of:

\[ 0 \leq p_s \leq 100 \]

### 2.1.2 Dependent variable

- **Packing density** is the fraction of the space filled by the particles making up the packing:

\[
\emptyset = \frac{V_p}{V_s} = \frac{V_s - V_v}{V_s} = 1 - \frac{V_v}{V_s} \quad (d)
\]

Where \( \emptyset \) is the packing density, \( V_p \) is the volume of the particles making up the packing, \( V_S \) is the volume of the total space of the packing, and \( V_v \) is the volume of the void space within the packing.
2.1.3 Controlled variable

- 3 types of particles were used in the experiment:
  - marble (of the same brand and material);
  - soybeans (near spherical particles, of the same brand and bag purchased); and
  - green beans (near spherical particles, of the same brand and bag purchased).
- Same plastics cylinder where the random packing of particles were built;
- Same people who performed the experiment;
- Same packing method was used to build the random packing in the experiment;
- Same graduated cylinders were used to measure the volume of water;
- Same scale was used to weigh particles;
- Same set of sieves used to control the size of particles.

2.2 Materials and Equipment List

- 500 8mm marbles;
- 1 kg soy beans;
- 1 kg green beans;
- One two litre plastic cylinder container;
- One 500 ml graduated cylinder;
- One 5 ml volumetric cylinder;
- One kitchen scale;
- One set of kitchen sieves;
- One rule;
- One candle;
- One metre of Aluminium foil;
- One flat plastic container;
- One hair dryer;
- One roll of paper towels;
- One bag of Coles Action Wipes – 10 multi – surface cleaning cloth;
- A pair of scissors;
- Permanent marker;
- White board makers;
- 2 round cardboards;
- Pencils;
- Paper;
- Matches
- Tap water.

2.3 Experimental Methods

2.3.1 Procedures

The experiment was to measure the packing density of randomly close packed mixture of binary particles, with certain size ratios and pre-defined number percentages of the small component in the mixture.

The two types of components used to build the binary mixtures in this study were either marbles and soybeans, or marbles and green beans. The marbles used in the experiment were always mono-sized spherical particles. The soybeans and green beans, however, were near spherical particles; and their sizes were controlled using sieves so that single sized soybeans and green beans were used. These resulted in two distinct size ratios obtained, one for the mixture of marbles and soybeans, and another for the mixture of marbles and green beans. Since marbles were bigger than both soybeans and green beans, the beans would act as the small component of the binary systems. Then the size ratios were calculated using the average radius of the soybeans to the radius of the marble, or the average radius of green beans to the radius of marble.

Similarly, the number percentages used in the study were derived from the number of beans in the mixture to the total number of particles of the mixture.

For a given size ratio and number percentage, packing density was obtained by comparing the volume of the void space to the total space of the packing itself using Equation (d).

To do so, for a desired size ratio and number percentage:

1) $N_s$ beans and $N_t$ balls were firstly selected.
They were then mixed evenly into a 2 litre plastic cylinder container. The mixture within the container was then shaken and compressed closely to create a random close packed structure.

A hard round cardboard was then put on the top of the mixed bed to record the height of the mixture from the bottom of the container.

The height of the mixture was measured using a ruler from the four cardinal directions, namely north, east, south, and west around the container. The height values obtained, notated as $H_n$, $H_e$, $H_s$, and $H_w$ respectively, were then averaged to obtain an average height of the mixture $H_a$.

The weight of the mixture together with the container was measured using a kitchen scale with an accuracy of gram (g) as its unit of measurement.

Tap water was then gradually poured into the mixture to fill in the void space of the packing. Once the water level reached the average height $H_a$ on the container, the new weight of the mixture together with the container and water was measured using the kitchen scale again. The weight difference comparing to the value obtained in step 5) would represent the volume of the tap water, which is also the volume of the void space $V_v$ of the packing with an accuracy of millilitre (ml).

The container was then emptied, dried and weighted again using the same scale.

Afterwards, tap water was poured into the empty container until the water level reached the same height $H_a$. The container (with the water) was then re-weighed using the scale. The weight difference compared with the weight of the empty container would give the volume of the total space of the packing $V_s$ itself as 1g water used would indicate a volume of 1 ml packing space.

Using Equation (d), packing density $\phi$ could be obtained for the designated size ratio and number percentage.

After drying the particles, steps 1) to 9) were repeated to ensure accuracy and reliability.

After drying the particles, steps 1) to 9) were repeated to measure the packing density for another packing with a different size ratio and/or number percentage.

The above obtained results were recorded in tables and graphs for further analysis.

2.3.2 Pre-treatment of the soy beans
As illustrated in the Procedure above, determination of packing density was achieved by measuring the volume of water for either the porous space of the packed bed or the space of the packing itself. Although water does not affect the size and shape of the marbles, it would affect those of the beans. To minimise the effect of water on the size and shape of the beans, thus the volume of the binary mixture, pre-treatment of the beans by waxing the surface of the beans became necessary. In this experiment, it was found that water has no noticeable effect on the size and shape of green beans, it however presented significant changes on the size and change of soybeans. As a result, the wax treatment of the soybeans was performed, which involved the following steps:

1) Large amount of wax scrap was obtained from a candle;  
2) Mixture of wax scrap and soybeans were placed into a flat plastic container lined with aluminium foil;  
3) The mixture was gradually heated using a hair dryer and shaken well and cooled down by air so that the surface of the beans started to be covered with a layer of wax.  
4) The heat, shake and cool process continued so that:  
   o Wax scrap was gradually melted and coated on the surface of the soybeans;  
   o Yet the beans still moved freely within the container, indicating only a thin layer of wax was coated;  
   o The extra wax was gradually pushed toward the boundary of the container and cooled together there with the help of aluminium foil.  
5) The beans were then removed from the container and poured onto the surface of a piece of cleaning cloth;  
6) The beans were gradually heated again using a hair dryer and stirred in the cleaning cloth and cooled so that extra wax except for the thin layer coated on the beans was further removed and absorbed by the cleaning cloth.

After wax treatment, water resistant soybeans were made for the experiment.

In the below sections, soybeans refer to the water resistant soybeans unless specifically mentioned otherwise.

2.3.3 Determination of the radii of the particles
To determine the size ratio of the packing components, radii of the particles used in the experiment, being either marbles, or soybeans or green beans, were measured separately using following steps:

1) Certain number of particles \( N \) was firstly counted and placed into a plastic cylinder container; the combined weight was measured using the scales.

2) Tap water was added into the plastic container until all these \( N \) particles were sunk under the water; and the new combined weight was measured using the scales.

3) The volume of the water used was recorded as \( V_1 \), calculated by the difference in weight between the final weight of the container at the end of Steps 1) and 2); and the water level was recorded as \( H \);

4) The particles were then removed from the plastic container, and the container was dried thoroughly using cleaning cloth and a hair dryer; and the empty container was weighed.

5) Tap water was gradually added into the plastic container until the same water level \( H \) was reached; and the combined weight was measured and recorded.

6) The volume of the water used this time was recorded as \( V_2 \); calculated by the difference in weight between the final weight of the container at the end of Steps 4) and 5).

7) The total volume of these \( N \) particles \( V \) was then obtained as:

\[
V = V_2 - V_1
\]  
(e)

8) The mean radius of these \( N \) particles \( r \) was derived as below:

\[
V = V_2 - V_1 = N \cdot \frac{4}{3} \pi r^3
\]

\[
\therefore r = \sqrt[3]{\frac{3(V_2-V_1)}{4\pi N}}
\]  
(f)

(g)

9) Steps 1) – 8) were repeated for marbles, soybeans and green beans, and radii of these 3 types particle were obtained.

For soybeans and green beans, the radius obtained would be the equivalent spherical radius respectively \(^{[22]}\).
2.3.4 Counts of the particles

To build a mixture of a binary packing, certain numbers of large and small particles were required. The counts of the large or the small particles could be obtained by using the weight of the large or the small particles divided by their average particle weight respectively.

To obtain the average particle weight of a certain type of particles, being either the marbles, or the soybeans, or the green beans used in this experiment, following steps were performed:

1) Certain number of a type of particles \( N \) was firstly counted and placed into a plastic container;
2) These particles were then weighted using a kitchen scale, after the tear weight of the container was waive off from the scale. The total weight of the particles was recorded as \( W \);
3) The average particle weight \( W_p \) of the measured type was obtained as below:
   \[
   W_p = \frac{W}{N} \tag{h}
   \]
4) Steps 1) to 3) were repeated for marbles, soybeans and green beans separately, so the average particle weights of these 3 types of particles were obtained.

3 Results

3.1 Particle attributes

After performing the steps listed in Sections 2.3.3 and 2.3.4, particle attributes of marbles, soybeans and green beans were obtained as recorded in Table 2.

Table 2. Particle Attributes

<table>
<thead>
<tr>
<th>Particle Type</th>
<th>Measured</th>
<th>Derived</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Particle Number ( N )</td>
<td>( V_1 ) (ml)</td>
</tr>
<tr>
<td>Marble</td>
<td>420</td>
<td>697</td>
</tr>
<tr>
<td>Soybean</td>
<td>735</td>
<td>108</td>
</tr>
<tr>
<td>Green bean</td>
<td>1600</td>
<td>61</td>
</tr>
</tbody>
</table>
3.2 Packing attributes

Using the marbles, soybeans and green beans, below two types of binary packing were studied:

I. Marbles and soybeans with size ratio $\eta = \frac{r_s}{r_l} = \frac{0.357}{0.790} = 0.452$; and

II. Marbles and green beans with size ratio $\eta = \frac{r_s}{r_l} = \frac{0.239}{0.790} = 0.302$.

By adjusting the particle numbers, below packing attributes were obtain in Table 3:
<table>
<thead>
<tr>
<th>Packing Type</th>
<th>Net Total Weight of Small Particles (g)</th>
<th>Particle Number</th>
<th>Height of the Packing Bed (mm)</th>
<th>Volume (ml)</th>
<th>Packing Density $\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small particles: Soybeans</td>
<td>-</td>
<td>420</td>
<td>0</td>
<td>0.0</td>
<td>115</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>420</td>
<td>47</td>
<td>10.1</td>
<td>115</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>420</td>
<td>105</td>
<td>20.0</td>
<td>114</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>420</td>
<td>180</td>
<td>30.0</td>
<td>113</td>
</tr>
<tr>
<td></td>
<td>Soybeans</td>
<td>68</td>
<td>420</td>
<td>281°</td>
<td>40.1</td>
</tr>
<tr>
<td></td>
<td>Large particles: Marbles</td>
<td>102</td>
<td>420</td>
<td>420°</td>
<td>50.0</td>
</tr>
<tr>
<td></td>
<td>Marbles</td>
<td>145</td>
<td>400</td>
<td>600°</td>
<td>60.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>145</td>
<td>257</td>
<td>600°</td>
<td>70.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>145</td>
<td>150</td>
<td>600°</td>
<td>80.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>145</td>
<td>67</td>
<td>600°</td>
<td>90.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
<td>0</td>
<td>735</td>
<td>100.0</td>
</tr>
<tr>
<td>Packing Type</td>
<td>Net Total Weight of Small Particles (g)</td>
<td>Particle Number</td>
<td>Height of the Packing Bed (mm)</td>
<td>Volume (ml)</td>
<td>Packing Density ( \phi )</td>
</tr>
<tr>
<td>--------------</td>
<td>----------------------------------------</td>
<td>----------------</td>
<td>-------------------------------</td>
<td>-------------</td>
<td>------------------</td>
</tr>
<tr>
<td>Type II</td>
<td></td>
<td>Large Particle</td>
<td>East ( H_e ), South ( H_s ), West ( H_w ), North ( H_n ), Average ( H_a )</td>
<td>Void ( V_v ), Packing Space ( V_s )</td>
<td></td>
</tr>
<tr>
<td>Small particles: Green beans</td>
<td>-</td>
<td>420</td>
<td>0</td>
<td>115</td>
<td>117</td>
</tr>
<tr>
<td>-</td>
<td>420</td>
<td>47</td>
<td>10.1</td>
<td>116</td>
<td>113</td>
</tr>
<tr>
<td>-</td>
<td>420</td>
<td>105</td>
<td>20.0</td>
<td>113</td>
<td>114</td>
</tr>
<tr>
<td>-</td>
<td>420</td>
<td>180</td>
<td>30.0</td>
<td>113</td>
<td>118</td>
</tr>
<tr>
<td>-</td>
<td>420</td>
<td>280</td>
<td>40.0</td>
<td>112</td>
<td>115</td>
</tr>
<tr>
<td>Large particles: Marbles</td>
<td>33</td>
<td>420</td>
<td>423°</td>
<td>50.2</td>
<td>113</td>
</tr>
<tr>
<td>49</td>
<td>420</td>
<td>628°</td>
<td>59.9</td>
<td>116</td>
<td>113</td>
</tr>
<tr>
<td>76</td>
<td>420</td>
<td>974°</td>
<td>69.9</td>
<td>114</td>
<td>112</td>
</tr>
<tr>
<td>131</td>
<td>420</td>
<td>1679</td>
<td>80.0</td>
<td>117</td>
<td>114</td>
</tr>
<tr>
<td>295</td>
<td>420</td>
<td>3782°</td>
<td>90.0</td>
<td>120</td>
<td>123</td>
</tr>
<tr>
<td>-</td>
<td>0</td>
<td>1600</td>
<td>100.0</td>
<td>40</td>
<td>41</td>
</tr>
</tbody>
</table>

Notes: (1) Values marked with * represent derived values according to the Average Particle Weight \( W_p \) and the Net Total Weight of the Small Particles in the packing.
(2) Results marked with underline _ indicate packing was made using a 250ml graduate cylinder, whereas all other packing was made using a 2 litre cylinder container.
The effect of size ratio and number percentage of small component on packing density is illustrated in Figure 7.

**Figure 7.** Effect of Size Ratio $\eta$ and Number Percentage of Small Component $P_s$ on Packing Density $\phi$ of binary particle packing.

4 Discussion

4.1 Aim and Hypothesis

The aim of the experiment was to investigate the effect of size ratio and number percentage of small component on the packing density of binary packing of particles. This was achieved by studying the packed structure created using marbles mixed with near spherical particles, either soybeans or green beans, with various populations of the beans in the mixtures.
The hypotheses were that the random mixing of different sizes particles would result in a change of packing density comparing to the packing of mono-sized spherical particles; that the larger the variation of the particle sizes, that the higher the packing density would result; and that for a given size ratio, there would be an optimum percentage in terms of the number of the small particles in the binary mixture that would result in the highest packing density.

From Table 3 and Figure 7, we can see that packing of mono-sized spherical marbles presents a packing density of 0.555. As small components e.g. either soybeans or green beans were introduced into the packed mixture, packing density becomes higher than that of the packing of the single sized marbles. This can be understood that, even for the random close packing of single size spheres, there are gaps or porous in between of the spheres. In this experiment, the porous occupies around 44.5% of the space. As a result, although introducing of small component to the packing would disturb the existing randomly packed marbles, the small components would also take the space that used to be the gaps, thus result in a change of the packing density, which usually an increase of the packing density as shown in the results.

In addition, packing mixed with green beans presents a more significant increase on packing density than that of with soybeans. This indicates that size ratio plays a vital role on packing structure and its packing density. As shown in Figure 7, the experiment results indicate that the larger the size variation is, the higher the packing density achieves, which also agrees with the hypothesis. It is interesting to note that when the large component is around twice as large as the small component, the effect of the size ratio on the packing density is noticeable, but not dramatic. However, when the large component is around more than three times larger than the small one, the impact on packing density becomes significant. This could be explained considering the size ratio thresholds compositing either a tetrahedral or an octahedral site in a closest pack of mono-sized spheres as per the Kepler Conjecture discussed early [2].
As illustrated in Figure 8, for the closest packing structure of either FCC or HCP, there are two possible “holes” existing in the structure, i.e., a tetrahedral or an octahedral “cell”. Within the “cell”, a small sphere can sit comfortably by just touching the surrounding large spheres without disturbing the “outer” structure of the spheres at all. The critical size ratios between the small sphere and the large sphere are 0.225 for the tetrahedral structure or 0.414 for the octahedral structure [23].

In this experiment, size ratio of 0.45 and 0.3 were both used in building packing structures. When the size ratio used was 0.45, a ratio larger than both of the critical ratios of tetrahedral and octahedral structures, introducing soybeans as the small component would possibly depart the marbles away as the “holes” of marbles might not big enough to host the soybeans without losing its packing structure. Comparing to soybeans, when green beans were used as the small components, it presented a size ratio of 0.3, which is between the two thresholds values 0.225 and 0.414. This suggests that, although green beans were not small enough to take the smallest holes of tetrahedral structure, they could accommodate with the cells of octahedral structure, and possibly the random cells in RCP. This means that in the random close packed marbles and green beans, green beans may use the “otherwise porous space” more efficiently than the soybeans, thus as affect the packing density more significantly.

Furthermore, the experiment results also indicate the existence of an optimum number percentage of small component for a type of packing under a given size ratio. For example, for the packing of marbles with soybeans where size ratio was around 0.45, the maximum packing density was presented for the mixture when the number of soybeans was about 60%
of the total number of particles including both marbles and soybeans. For the packing of marbles and green beans, where the size ratio was around 0.30, the maximum packing density was achieved when the number of green beans was about 90% of the total number of particles. This agrees with our hypothesis and further supports the previous analysis that even with a number percentage as high as 90%, green beans may still act as a “filling” component that fit within the gaps of marbles and thus increase the packing density accordingly. This type of “adding small to large” mechanism continues from 0 to around 90% for green beans until the optimum point, and then the opposite “adding large to small” mechanism begins \[24\]. When the number percentage of green beans reached 100%, i.e. a pure packing created using no marbles, the packing density decreased to 0.603 as shown in Table 3 and Figure 7. This can be understood that the “adding large to small” effect eventually vanished so the space 100% filled by the marbles was all required to be fulfilled by the green beans where porous space among could no longer 100% filled. Similarly, the optimum point for the packing of marbles and soybeans can also be understood. However, due to the size ratio was not significant to indicate a sharp filling mechanism, the overall change of packing density was much less dramatic comparing to the packing of marbles and green beans.

4.2 Error analysis

4.2.1 Systematic errors

- Container

My experiment results suggest random close packed spherical particles i.e. the marbles, has a packing density of 0.555, which is lower than the value of 0.634 suggested in literature \[^{10}\]. This is probably due to the container size used in this study was not big enough to eliminate the boundary effect of the packing. Comparing to this, the packing densities obtained from packing of soybeans and green beans was 0.565 and 0.603. These were also lower than 0.634, although it may also due to the fact soybeans and green beans used were not 100% spherical.

While this remains a challenge in experiment, as what size of container would be acceptable for minimise the effect of container boundary, or how to obtain the “central” part of a packing where boundary effect was removed, the so-called Periodic Boundary Conditions in computer simulation technique could be a solution for this problem \[^{25}\].
• Non spherical beans vs. spheres
  In this study, soybeans and green beans were used as the particles to mix with marbles. Although these beans were round, they were not totally spherical particles like marbles. This introduced some uncertainty on the effect of particle shape on the packing density, because marbles, soybeans and green beans present different sphericity.

Without uniformed sphericity, it is uncertain to what degree the shape would affect the dependent variable packing density in this experiment.

• Measuring the particle sizes
  The ruler used in this experiment provides an accuracy of ±0.5mm. As shown in Table 2, the smallest mean particle radius was around 2.4mm. If a ruler was directly used to measure the particle, it would yield an inaccuracy rate of 21% when measuring the mean particle radius. Thus in the calculation of the size ratio, this inaccuracy will double to become 42%. To solve this problem, an alternate method, introduced in Section 2.3.3, was used. In the alternate method, the volume taken up by the particles were used to determine their radii. Because the absolute error of the scale was 1 gram, equating to 1 ml of volume, the relative error rate for Table 2 data is ±1.1%. The error therefore for the derived mean particle radius will be ±1.1% × 1/3 = ±0.4%, which would mean a ±0.8% inaccuracy in calculating the size ratio.

4.2.2 Random errors
• Measuring the volume
  The study of this experiment relies on how accurate the volumes, either of the porous space of the packing or the packing structure itself, were measured. Although the most accurate volumetric cylinder provides an accuracy of ±0.25 ml, considering that the 2 litre cylinder container where most of the packing was built had a wider diameter, the misjudging of the liquid meniscus height may introduce significant parallax errors when measuring the volumes.
On the other hand, because the generated packing may not always present a levelled surface top, determination of the height of the packed beds will affect the accuracy on the measurement of the void space and packing volume.

To solve the problem, when measuring the height of a packing, four measures were performed diagonally across the surface top. Then an average height was derived and used as the true height of the packing. This was found to help control the random error introduced when determining the height of a packing.

4.3 Improvements

To improve this experiment, the two possible emendations below could be made to the method in which the experiment was carried out:

- Suitable container determination by matching the packing density reached by using the large balls until the well-accepted packing density is replicated.

  This may need more containers to be tried out, and more marbles are necessary to carried out the experiment.

  Once the ideal container is identified, the remaining experiments could be performed using this container where boundary could be minimised.

- Replacing non-spherical particles e.g. beans with spherical particles.

  This would eliminate the effect of particle shape on the effect of packing density.

  Alternatively, replacing marbles and the beans with other “beans” with a wider range of sizes, yet retaining the same sphericity is also acceptable, but could be practically challengeable, if not entirely impossible.
5 Conclusion

The aim of the experiment was achieved. In this experiment, random binary particle packing built with marbles and soybeans, or glass green beans was studied experimentally. The effect of size ratio and number percentage of the small component on the packing density was investigated. It was conclusively determined that the random mixing of different sizes particles would result in a change in the packing density comparing to the packing of mono-sized spherical particles. Size ratio was a key factor that affected packing density. Experimental results indicated that the larger the variation of the particle sizes, the higher the packing density that resulted. For a given size ratio, an optimum number percentage of the small particles exists that results in the highest density of the packing.
6 Appendix – Photographs of Procedures

Figure 9. The 2 litre plastic container

Figure 10. Kitchen Scale used to measure weight

Figure 11. Waxing of the soybeans
Figure 12. Green beans as used in the experiment.

Figure 13. Marking Equipment used in the Experiment.

Figure 14. Round pieces of cardboard used to determine the height of the packed bed.
Figure 15. Soybeans used in the experiment.

Figure 16. Green beans used in the experiment.

Figure 17. Candle as used in the experiment.
Figure 18. Hairdryer as used in the experiment.

Figure 19. Example of a Random Close Packing.
Bibliography


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